

Constraints on non-Newtonian gravity from the Casimir force measurements between two crossed cylinders

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Constraints on the Yukawa-type corrections to Newtonian gravitational law are obtained resulting from the measurement of the Casimir force between two crossed cylinders. The new constraints are stronger than those previously derived in the interaction range between 1.5 nm and 11 nm. The maximal strengthening in 300 times is achieved at 4.26 nm. Possible applications of the obtained results to the elementary particle physics are discussed.

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Non-Newtonian gravity is the subject of a considerable amount of literature (see the monograph [1] and references therein). Corrections to Newtonian gravitational law at small distances are predicted by the unified gauge theories, supersymmetry, supergravity and string theory. They arise due to exchange of massless and light elementary particles (such as axion, scalar axion, arion, dilaton, graviphoton etc) between the atoms of two macrobodies [2]. Another reason for non-Newtonian gravity is the conceivable existence of extra spatial dimensions. Recent trends show an increase in use of Kaluza-Klein field theories with a weak compactification scale [3,4]. These theories predict that deviations from the Newtonian gravitational law exist at submillimeter range [5-7]. Namely in this range gravitational law is not confirmed experimentally (the common faith in its validity up to the Planckian length of about 10^{-33} cm is nothing more than a far-ranging extrapolation [8]).

Non-Newtonian gravity is usually described by the Yukawa- or power-type corrections to the Newtonian potential which is inversely related to distance. The constraints on the constants characterizing the magnitude and interaction range of these corrections are usually obtained from the Cavendish- and Eötvös-type experiments, Casimir and van der Waals force measurements. The gravitational experiments are the best in the interaction range $10^{-2} \text{ m} < \lambda < 10^6 \text{ km}$ (see the most strong constraints obtained by this way in [9]). At submillimeter range the best constraints on the Yukawa-type corrections to Newtonian gravity follow from the measurements of the Casimir and van der Waals forces [2,10,11]. Recently the new experiments on measuring the Casimir force were performed [12-15]. With the results of [12] the constraints on the Yukawa-type interaction were strengthened in [16] up to 30 times in the interaction range $2.2 \times 10^{-7} \text{ m} \leq \lambda \leq 1.6 \times 10^{-4} \text{ m}$ (see also [17]). By the use of [13-15] the constraints known to date were strengthened in [18-20] up to 4500 times in the interaction range $4.3 \times 10^{-9} \text{ m} \leq \lambda \leq 1.5 \times 10^{-7} \text{ m}$. There are also other experiments (see, e.g., [21]) but the constraints obtained from them are weaker than those mentioned above.

In paper [22] the Casimir force was measured between the gold surfaces of two crossed cylinders covered by the thin layers of hydrocarbon. Using a template-stripping method, the root mean square roughness of the interacting surfaces was decreased up to 0.4 nm. This gave the possibility to measure the Casimir force at the closest separation of 20 nm with a resolution $\sim 10^{-8} \text{ N}$. The decreased distance of closest separation provides a way for obtaining more strong constraints on the constants of Yukawa-type corrections to Newtonian gravity at small distances. As shown in the present paper, the constraints, following from [22], are up to 300 times stronger than the previously known ones in the interaction range $1.5 \times 10^{-9} \text{ m} \leq \lambda \leq 11 \times 10^{-9} \text{ m}$.

In order to calculate non-Newtonian force between two crossed cylinders we consider first two point masses M_1 and M_2 at a distance r_{12} apart. The effective potential of gravitational interaction with account of Yukawa-type corrections is [1,8,18,19]

$$V(r_{12}) = -\frac{GM_1M_2}{r_{12}} \left(1 + \alpha_G e^{-r_{12}/\lambda}\right). \quad (1)$$

Here G is Newtonian gravitational constant, α_G is a dimensionless interaction constant, λ is the interaction range. The gravitational potential with account of power-type corrections is of the form

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$$V_n(r_{12}) = -\frac{GM_1M_2}{r_{12}} \left[1 + \lambda_n^G \left(\frac{r_0}{r_{12}} \right)^{n-1} \right], \quad (2)$$

where $r_0 = 10^{-15}$ m is introduced to provide the proper dimensionality with different n [23], λ_n^G is the interaction constant depending on n . In both Eqs. (1), (2) the non-Newtonian gravity dominates at small separations. With increase of the separation the non-Newtonian corrections vanish comparing the Newtonian contribution.

The interaction potential acting between two crossed cylinders is obtained by the integration of Eqs. (1), (2) over their volumes. The end effect consists of two contributions — Newtonian one and Yukawa- or power-type one. The case of Yukawa-type interaction will be our initial concern. Newtonian gravitational interaction will be estimated next. Let the first cylinder of radius R_1 be arranged vertically so that its axis coincides with the axis z . Both cylinders can be considered as infinite because their lengths are much larger than their radii.

We consider an arbitrary point P belonging to the second cylinder and from P drop a perpendicular to the axis of the first cylinder. In a spherical coordinate system with center O_1 located at the intersection of the first cylinder axis and the above perpendicular the coordinates of point P are $(r_2, \theta_2 = \frac{\pi}{2}, \varphi_2 = \frac{\pi}{2})$. Here it is suggested that O_1P coincides with y axis and angle φ is counted from x axis. Distance r_{12} is now a separation between an arbitrary point of the first cylinder with coordinates $(r_1, \theta_1, \varphi_1)$ and a point P .

Interaction potential of Yukawa-type between the first cylinder and a point P can be obtained by the integration over the cylinder volume

$$\Phi(r_2) = -GM_1M_2\alpha_G N_1 \int_0^{2\pi} d\varphi_1 \int_0^\pi \sin \theta_1 d\theta_1 \int_0^{R_1/\sin \theta_1} r_1^2 dr_1 \frac{e^{-r_{12}/\lambda}}{r_{12}}, \quad (3)$$

where N_1 is the atomic density of the first cylinder material.

To calculate integrals in Eq. (3) let us take advantage of the expansion in spherical harmonics [24]

$$\frac{1}{r_{12}} e^{-r_{12}/\lambda} = 4\pi \sum_{l=0}^{\infty} a_l(r_1, r_2) \sum_{m=-l}^l Y_{lm}^*(\theta_1, \varphi_1) Y_{lm}(\theta_2, \varphi_2), \quad (4)$$

where the coefficients are expressed in terms of Bessel functions of imaginary argument

$$a_l(r_1, r_2) = \frac{1}{\sqrt{r_1 r_2}} I_{l+\frac{1}{2}} \left(\frac{r_{<}}{\lambda} \right) K_{l+\frac{1}{2}} \left(\frac{r_{>}}{\lambda} \right), \quad (5)$$

and $r_{<} \equiv \min(r_1, r_2)$, $r_{>} \equiv \max(r_1, r_2)$.

Now we take into account that the radii of the cylinders in [22] are rather large ($R_1 = R_2 = R = 1$ cm) comparing the values of λ for which the strong constraints are obtainable at separations of about 20 nm. It follows from here, that not only $\lambda \ll r_{>}$ but also $\lambda \ll r_{<}$ (since only a thin layer adjacent to the first cylinder surface contributes to the force). As a result the conditions $r_{<}/\lambda, r_{>}/\lambda \gg 1$ are valid and one may use the asymptotic expressions for Bessel functions of large arguments

$$I_{l+\frac{1}{2}}(z) \approx \frac{1}{\sqrt{2\pi z}} e^z, \quad K_{l+\frac{1}{2}}(z) \approx \sqrt{\frac{\pi}{2z}} e^{-z}. \quad (6)$$

Substitution of (6) into (5) in the case of $r_1 < r_2$ leads to the following expression

$$a_l(r_1, r_2) = \frac{\lambda}{2r_1 r_2} e^{(r_1 - r_2)/\lambda}. \quad (7)$$

For $r_1 > r_2$ one obtains

$$a_l(r_1, r_2) = \frac{\lambda}{2r_1 r_2} e^{(r_2 - r_1)/\lambda}. \quad (8)$$

Note that in both cases the value of a_l under the above conditions does not depend on l . This gives the possibility to calculate explicitly all integrals in Eq. (3). To do this we substitute (4), (7), (8) into (3) and use the completeness relation for spherical harmonics [24]

$$\sum_{l=0}^{\infty} \sum_{m=-l}^l Y_{lm}^*(\theta_1, \varphi_1) Y_{lm}(\theta_2, \varphi_2) = \delta(\varphi_1 - \varphi_2) \delta(\cos \theta_1 - \cos \theta_2) \quad (9)$$

at a point $\theta_2 = \varphi_2 = \frac{\pi}{2}$. The result is

$$\Phi(r_2) = -2\pi G \rho_1 M_2 \alpha_G R_1 \lambda^2 e^{R_1/\lambda} \frac{1}{r_2} e^{-r_2/\lambda}, \quad (10)$$

where $\rho_1 = M_1 N_1$ is the density of the first cylinder.

In order to calculate the interaction potential between the two cylinders it is necessary now to integrate Eq. (10) over the volume of the second cylinder which is aligned perpendicular to the first one at the closest separation a . Thus, the separation between the axes of the cylinders is $R_1 + R_2 + a$. We consider now the cylindrical coordinate system. Let z axis of it coincides with the axis of the first cylinder and let the origin be at a point nearest to the axis of the second cylinder. In this coordinate system the coordinates of any point belonging to the second cylinder are $(\tilde{r}_2, \tilde{\varphi}_2, \tilde{z}_2)$. Here \tilde{r}_2 is the separation between the point of the second cylinder and the axis of the first one.

Interaction potential between the two cylinders is given by

$$U(a) = 4N_2 \int_0^{\pi/2} d\tilde{\varphi}_2 \int_0^{R_2} d\tilde{z}_2 \int_{\rho_{\min}(\tilde{z}_2, \tilde{\varphi}_2)}^{\rho_{\max}(\tilde{z}_2, \tilde{\varphi}_2)} d\tilde{r}_2 \tilde{r}_2 \Phi(\tilde{r}_2). \quad (11)$$

Here the function Φ is defined in (10) and

$$\rho_{\max(\min)}(\tilde{z}_2, \tilde{\varphi}_2) = \frac{R_1 + a + R_2 \pm \sqrt{R_2^2 - \tilde{z}_2^2}}{\cos \tilde{\varphi}_2}. \quad (12)$$

Calculating integrals in (11) with the help of [25] and using the conditions $\lambda, a \ll R_1, R_2$ one obtains

$$U(a) = -4\pi^2 G \rho_1 \rho_2 \alpha_G \lambda^4 \sqrt{R_1 R_2} e^{-a/\lambda}, \quad (13)$$

where ρ_2 is the density of the second cylinder.

The Yukawa-type force acting between the two cylinders is obtained from (13)

$$F_Y(a) = -\frac{dU(a)}{da} = -4\pi^2 G \rho^2 \alpha_G \lambda^3 \sqrt{R_1 R_2} e^{-a/\lambda}. \quad (14)$$

Exactly the same expression can be obtained from the Yukawa-type energy of two plane parallel plates by the application of proximity force theorem [26]. As for the power-type interactions, distinct results are obtained with different n . By way of example, with $n \geq 5$ the power-type force acting between two crossed cylinders can be represented as

$$F_n(a) = -\frac{4\pi^2 r_0^{n-1} G \lambda_n^G \rho^2}{(n-2)(n-3)(n-4)} \frac{R}{a^{n-4}}. \quad (15)$$

In the experiment [22] two identical cylinders were made of the same material and covered by the thin layers of gold (of thickness Δ_1 and density ρ') and hydrocarbon (thickness Δ_2 , density ρ''). The force acting in experimental configuration can be easily obtained by the combination of several expressions of Eqs. (14), (15). For the Yukawa-type force the result is ($\rho = \rho_1 = \rho_2$)

$$F_Y(a) = -4\pi^2 G \alpha_G \lambda^3 R e^{-a/\lambda} \left[\rho'' \left(1 - e^{-\Delta_2/\lambda} \right) + \rho' \left(1 - e^{-\Delta_1/\lambda} \right) e^{-\Delta_2/\lambda} + \rho e^{-(\Delta_1+\Delta_2)/\lambda} \right]^2. \quad (16)$$

Returning back to the Newtonian gravitational force, given by the first term in the right-hand side of Eqs. (1), (2), we find it negligible to compare with the Casimir force at the closest separations. Actually, the gravitational force acting between the crossed cylinders spaced at $a \ll R$ apart can be roughly estimated as the force acting between two spheres of radius R

$$F_N \sim G \frac{\left(\frac{4}{3} \pi \rho R^3 \right)^2}{4R^2} = \frac{4}{9} \pi^2 G \rho^2 R^4. \quad (17)$$

If the cylinders are made of quartz with $\rho = 2.23 \times 10^3 \text{ kg/m}^3$ the Newtonian gravitational force is estimated as $F_N \sim 1.4 \times 10^{-11} \text{ N}$. The Newtonian force is rather small, even though the cylinders as a whole are made of gold with $\rho' = 18.88 \times 10^3 \text{ kg/m}^3$. In this case $F_N \sim 1 \times 10^{-9} \text{ N}$ which is still much less than the Casimir force at the separations of 20 nm equal to $F_C \approx 9.4 \mu\text{N/m} \times 2\pi R \approx 6 \times 10^{-7} \text{ N}$ [22]. As a consequence one should not add the contribution of Newtonian gravity to the non-Newtonian terms given by Eqs. (14), (15). Newtonian gravity can be neglected when obtaining constraints on hypothetical interactions from the results of Casimir force measurements at smallest separations.

We are coming now to the obtaining constraints on the Yukawa-type interaction. According to the result of [22] the theoretical expression for the Casimir force acting between two crossed cylinders was confirmed within the limits of experimental error of force measurement $\Delta F = 10 \text{ nN}$. This theoretical expression was obtained with regard to the corrections due to the finite conductivity of the boundary metal, stochastic roughness, covering hydrocarbon layer, and nonzero temperature (in fact temperature corrections are insignificant at the separations $a < 100 \text{ nm}$ considered in [22]). The relative accuracy of force measurements at the smallest separations can be estimated as $\delta = \Delta F/F_C \approx 1.7\%$. It increases quickly, however, with increasing of a .

Since no any Yukawa-type interaction was observed in the limits of experimental error, it is evident that

$$|F_Y(a)| \leq \Delta F, \quad (18)$$

where $F_Y(a)$ is defined by Eq. (16).

The constraints on the parameters of Yukawa-type interaction α_G and λ are obtained from Eq. (18) by substituting the values of quartz and gold densities given above and also the density of hydrocarbon layer $\rho'' \approx 0.85 \times 10^3 \text{ kg/m}^3$. The thicknesses of the layers are $\Delta_1 = 200 \text{ nm}$, $\Delta_2 = 2.1 \text{ nm}$. The strongest constraints follow at the closest separation distance $a = 20 \text{ nm}$. The computational results are shown in Fig. 1 by the curve 1 at the logarithmic scale. In the same figure the curves 2 and 3 show the constraints which follow [19,20] from the Casimir force measurements between a plane disk and a spherical lens by means of atomic force microscope [14,15]. Curve 4 presents the constraints obtained from the Casimir force measurements between dielectrics [2]. Curve 5 shows constraints following from the measurements of the van der Waals force [10]. The regions above the curves 1–5 are prohibited by the results of the corresponding experiment. The regions below the curves are permitted.

As seen from Fig. 1, the Casimir force measurements of [22] lead to strongest constraints on the constants of Yukawa-type corrections to Newtonian gravitational law within the interaction range $1.5 \text{ nm} < \lambda < 11 \text{ nm}$. The largest strengthening in 300 times is achieved at $\lambda = 4.26 \text{ nm}$. For $\lambda < 1.5 \text{ nm}$ the best constraints follow from the measurement of the van der Waals force. For $11 \text{ nm} < \lambda < 150 \text{ nm}$ the constraints from the Casimir force measurements between gold surfaces of a spherical lens and a disk [15] are the strongest ones.

According to Fig. 1, at nanometer range the Yukawa-type corrections are still permitted by the experiment which are in excess of the Newtonian gravitational interaction of more than 20 orders of magnitude. The nanometer range corrections to Newtonian gravity are of especial interest for the weak-scale compactification schemes with three extra spatial dimensions (total space-time dimensionality $N = 7$). In fact, in this case the compactification dimension is equal to [27]

$$R \sim 10^{\frac{30}{N-4}-19} \text{ m} = 10^{-9} \text{ m} = 1 \text{ nm}, \quad (19)$$

and at the several times larger separations the non-Newtonian gravity should be noticeable. Exchange of the hypothetical particles mentioned above also may contribute to the Yukawa-type force. That is the reason why the further strengthening of constraints on the corrections to Newtonian gravity in nanometer range presents interest for the elementary particle physics.

It is easy to check that no strong constraints are obtainable from the experiment [22] on the constants of power-type corrections to Newtonian gravity. For $n = 5$, as an example, the constraint following from Eq. (15) and an inequality

$$|F_5(a)| \leq \Delta F = 10 \text{ nN} \quad (20)$$

is $|\lambda_5^G| \leq 0.92 \times 10^{49}$. This is much weaker than $|\lambda_5^G| \leq 2.1 \times 10^{47}$ obtained from the Cavendish-type experiments [28]. As noted in [16], the Casimir force measurements between metallic surfaces produce weaker constraints on the power-type interactions than the gravitational experiments (see [9] for the latest results).

As was shown above, the Casimir force measurement between the metallized surfaces of two crossed cylinders gives the possibility to strengthen constraints on the non-Newtonian gravity of Yukawa-type up to 300 times in the nanometer range. During the last few years several new measurements of the Casimir force between metallic bodies were performed [12–15,22] and each of them gave possibility to obtain more strong constraints on the constants of Yukawa-type corrections to the Newtonian gravitational law [16–20]. Notice that all these experiments were designed for the registration of the Casimir force only. There were no opportunities used especially in order to increase the

sensitivity of an experiment to the probable Yukawa-type interaction which were proposed in the literature (see, e.g., [2,8]). This means that the potentialities of the Casimir force measurements have not been exhausted.

Currently several new experiments on the measurement of the Casimir force are in preparation. They will give possibility to strengthen further the constraints on non-Newtonian gravity up to 10^4 times in a wide interaction range from 10^{-9} m till 10^{-3} m. As a result the values of $\alpha_G < 0.1$ will be achieved near the right boundary of this interval (the method using the dynamical Casimir force [21] is also high promising in the process). In this way the Casimir effect is quite competitive with the modern accelerators and also with the Eötvös- and Cavendish-type experiments in the search for light elementary particles and hypothetical long-range interactions predicted by the modern theories of fundamental interactions.

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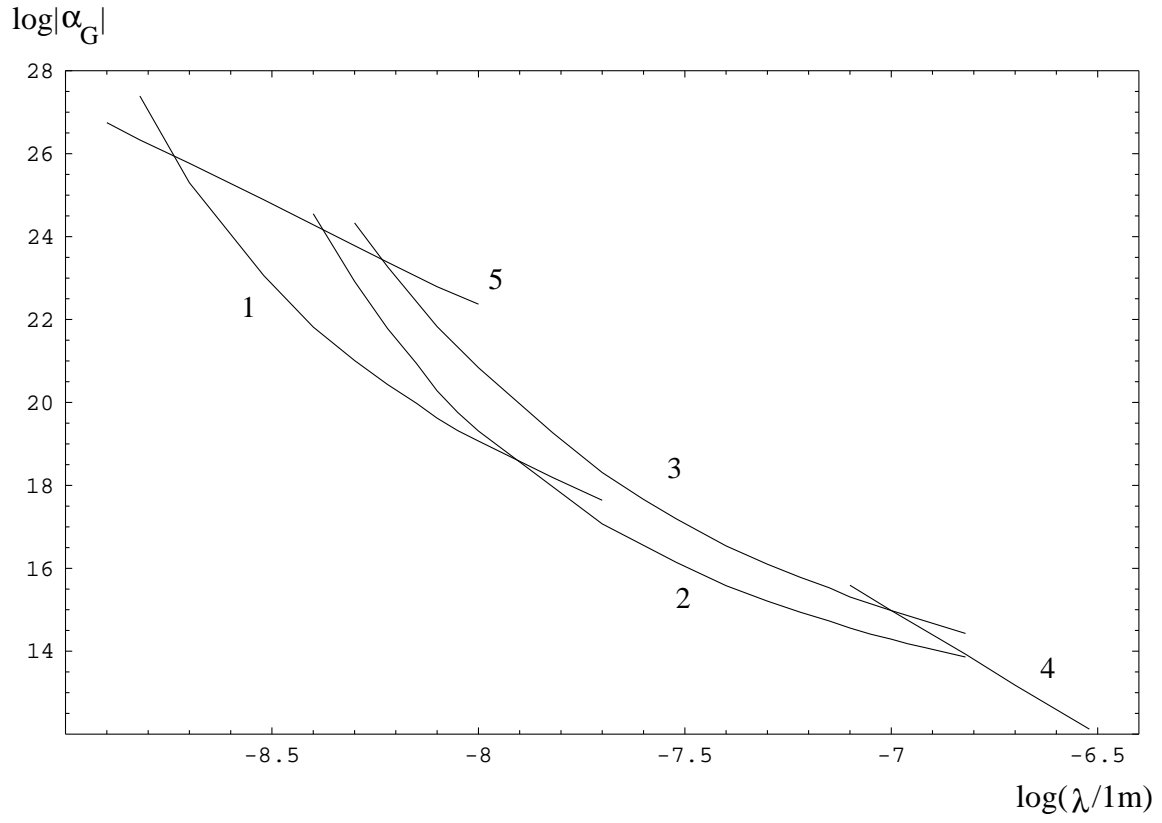


FIG. 1. Constraints on the constants of Yukawa-type corrections to Newtonian gravity. Curves 4 and 5 were obtained from the measurements of the Casimir and van der Waals forces respectively between dielectrics. Curves 2 and 3 follow from the Casimir force measurements between gold and aluminum surfaces by means of atomic force microscope. Curve 1 is obtained in this paper from the Casimir force measurement between two crossed cylinders. The regions below the curves are permitted, and those above the curves are prohibited.